Limits to Firm Growth: All in the Family?

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This Version: November 8, 2021

Preliminary and Incomplete

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Abstract

In less developed countries, firms tend to be small and many are family firms. We build a model of joint production in which managers collaborate subject to limited contract enforceability. Such contractual frictions keep firms small and give rise to family firms because collaboration among family members is better sustained than among professional managers. However, family members have different productivities, which is a source of disadvantage due to complementarity in joint production. The degree of contract enforceability and families' size and productivity endowment determine the prevalence of single manager firms, family firms (with or without outside managers), and professional firms in the economy, as well as the firm size distribution and aggregate productivity. Our quantitative model based on Indian micro data shows that India's income per capital would be 7 to 16 percent higher if contracts in India were enforced as well as in the US. If family firms are not allowed in the model, this income gap increases by 14 to 20 percent, since family firms are a way of mitigating the contractual frictions. Dissolving all family firms results in an income loss of 1 to 3 percent to large wealthy families and small poor families. In addition, the mid-range of the firm size distribution hollows out and income inequality worsens. Finally, a policy reducing family sizes undermines the role of family firms in mitigating the impact of contractual frictions and hence reduces income per capita, which contrasts with the conventional wisdom on fertility and economic development.

Keywords: Family Firms, Contracting Frictions, Firm Growth

JEL Classification: O4, J24, L1

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The authors thank the participants at the Bank of Italy/ CEPR/ EIEF Conferenfce for helpful comments and discussions. The views expressed herein are solely those of the authors and do not necessarily reflect those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

1. Introduction

There are substantial differences in per-capita income across countries, driven primarily by total factor productivity (TFP). For instance, GDP per capita of India in 2018 was a little over $2,000 compared to about $60,000 for the US. At the same time, literature has documented important differences in organization of production across countries.

In less developed countries such as India, there are smaller firms and establishments with substantially lower growth compared to firms in the US (see for example, Tybout (2000), Hsieh and Klenow (2014), Hsieh and Olken (2014)) resulting in aggregate productivity loss. There is also evidence of centralization of decisions within firms in developing countries, as a result of lack of trust, leading to lower aggregate productivity, as documented by Bloom, Sadun, and Van Reenen (2012). Another feature of developing countries is the prominence of family firms. Literature has documented that generally family firms are less productive than their non-family counterparts (see for example, Bertrand and Schoar (2006) and Bertrand, Johnson, Samphantharak, and Schoar (2008) for a review of the literature documenting this). Another strand of literature has documented cross-country differences in the rule of law and contract enforcement and its implications on shareholders and ownership (Porta, Lopez-de Silanes, Shleifer, and Vishny (1998)).

These different perspectives on the constraints to firm growth and the prevalence of family firm in poor countries raise the natural questions: (i) what is the impact of these constraints in driving per-capital income differences across countries; (ii) what is the role of family firms in ameliorating or exacerbating these constraints?

In particular, we analyze the drivers of the substantial differences in firm-size distribution across India and the US as seen clearly in figure 1? More specifically, we ask how does weak rule of law and enforcement affects the organization of production in the less developed countries whereby we see smaller establishments, more centralization of decisions and existence of family firms? Lastly, and most importantly, what are the implications of weak enforcement for aggregate productivity and output. To this end, we develop a model of firms as a collection of managers and workers where managers have heterogeneous productivity and there exists complementarity in production function. The model features increasing returns to the number of managers or gains from specialization à la Becker and Murphy (1992).

Our model gives rise to the existence of a fairly rich set of firms: single person firms, professional firms, family firms without any outside managers and family firms with outside managers. In our model, the size of any non-family firm is limited by the ability of managers to divert a fraction of output, i.e., imperfect enforcement of contracts. On the other hand, size of a family firm is limited by the number of family members and their productivity endowment. This gives rise to a large number of small and unproductive firms and a few very large and productive professional firms.

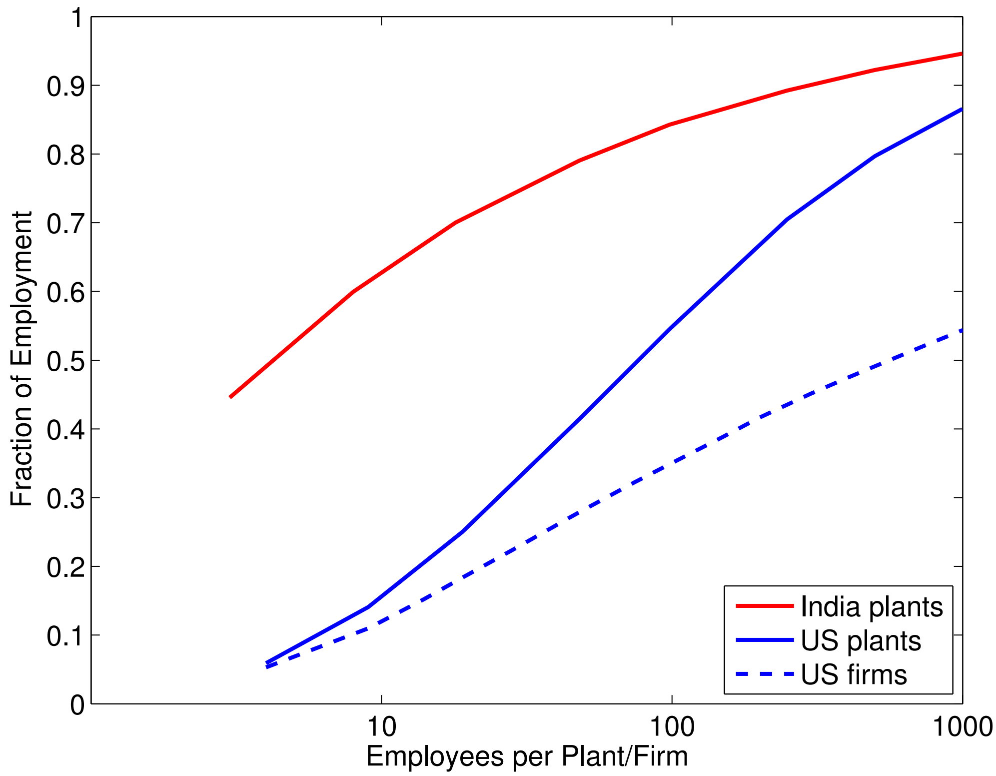


Figure 1: Size Distribution of Plants (Firms)

We estimate out model on the firm and household level micro-data from India. Using our estimated model, which includes the estimates for delegation friction in India, we back out the delegation friction in the US by matching the average firm size in the US. We find that India's income per capital would be 7 to 16 percent higher if contracts in India were enforced as well as in the US. Further, if family firms are not allowed in the model, this income gap increases by 14 to 20 percent, since family firms are a way of mitigating the contractual frictions. Dissolving all family firms results in an income loss of 1 to 3 percent to large wealthy families and small poor families. In addition, the mid-range of the firm size distribution hollows out and income inequality, and firm concentration worsens. Finally, a policy reducing family sizes undermines the role of family firms in mitigating the impact of contractual frictions and hence reduces income per capita, which contrasts with the conventional wisdom on fertility and economic development.

Related Literature: This paper builds on and contributes to several strands of the literature. First, It contributes to the literature on organization of production across countries and with the level of development, such as Bloom, Sadun, and Van Reenen (2012), Hsieh and Olken (2014), Bloom and Van Reenen (2010), Tybout (2000), Hsieh and Klenow (2014), Hopenhayn (2016), and Poschke (2018). On cross-country differences in the rule of law and contract enforcement, such as Porta, Lopez-de Silanes, Shleifer, and Vishny (1998). On the role of managers, it relates to the empirical findings by Bloom, Eifert, Mahajan, McKenzie, and Roberts (2013). It contributes to the literature on family firms, such as Burkart, Panunzi, and Shleifer (2003),Bertrand and Schoar (2006), Bertrand, Johnson, Samphantharak, and Schoar (2008), Bhattacharya and Ravikumar (2004), Lemos and Scur (2019). Closest paper to ours is Akcigit et al. (2016), we differ for them by explicitly modeling firms in presence the contract enforcement which endogenously give rise to family firms. Lastly, it contributes to the literature quantifying the value of family firms, such as Atkeson and Irie (2020) who look at the wealth accumulation.

We start with some empirical facts across the level of development in section 2, followed by the model, data and quantitative exercise in section 3, 4 and 5 respectively.

2. Empirical Facts

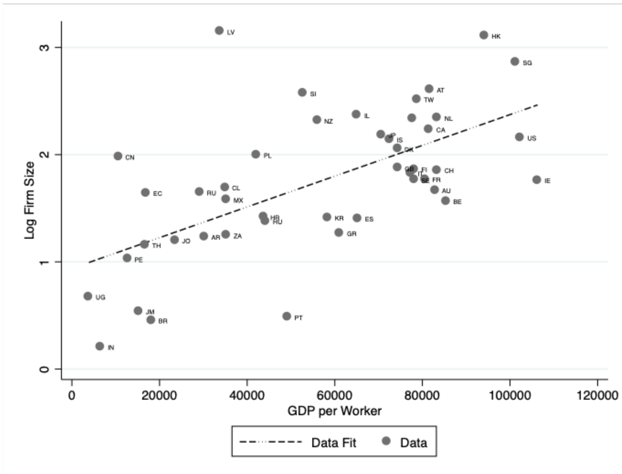
We describe the key empirical facts on firm size, family firms and contract enforcement across the level of development in this section. Data sources are detailed in the data appendix.

2.1 Poorer countries have smaller firm size compared to rich

Countries with higher output per capita, on average, have larger firms. This robust relationship which holds across the level of development. In figure 2, we plot log firm size by output per worker across countries using Global Entrepreneurship Management and Penn World Table data. For example, India (denoted by IN) is at the bottom end of the development spectrum shown in the bottom left of the figure with $6,600 output per worker and firm size of 2.7 while the US has $102,417 output per worker and average firm size of 11.5[1].

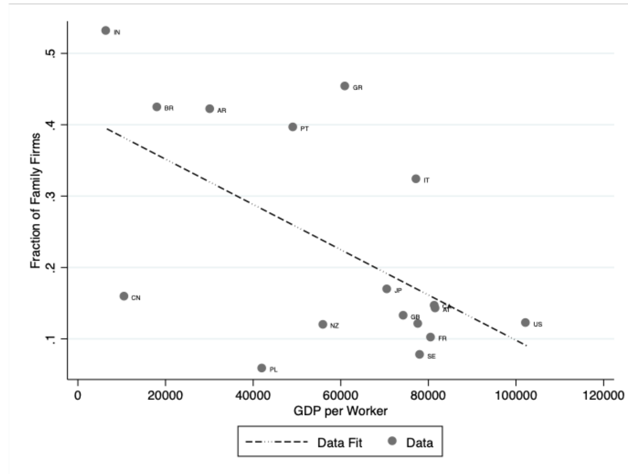
[1]Note: GEM firm size is smaller than other data sources, see for example, Poschke (2018) for a discussion on this. We use US census aggregate data in our delegation friction exercise.

Figure 2: Firm Size and GDP



Source: GMS and PWT

Figure 3: Family Firms and GDP



Source: WMS and PWT

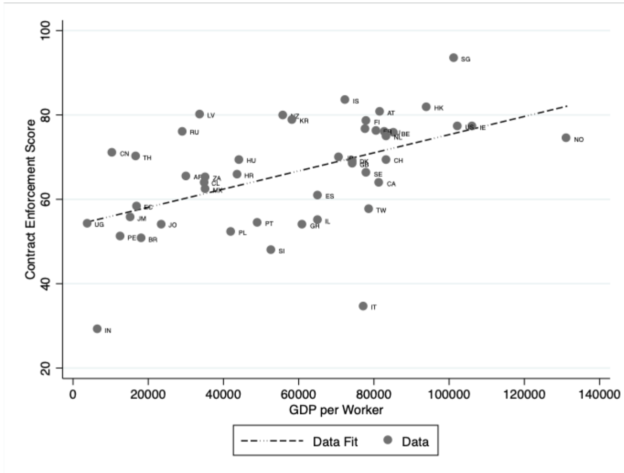
2.2 Poorer countries have a higher fraction of family firms firm size compared to rich

At the same time, poorer countries, such as India also feature a higher fraction of family firms, a fraction that goes down for developed countries such as the US (0.12 vs 0.52 of India). In figure 3, we plot fraction of family firms by output per worker across countries using World Management Survey and Penn World Table. Family firms are identified using the micro-data as specified in Appendix A.1.

2.3 Poorer countries have weaker contract enforcement compared to rich

Using contract enforcement score from World Bank ease of doing business report, we find that poor countries such as India have a lower contract enforcement compared to the developed countries such as the US which have stronger enforcement of laws. In figure 4, we plot the contract enforcement score by output per worker across countries using World Bank data and Penn World Table. India's contract enforcement score is 29 compared to 77 for the US.

Figure 4: Contract Enforcement and GDP



Source: World Bank and PWT

Motivated by the empirical facts about firm size, family firms and contract enforcement, we describe our model in the following section.

3. Model

Our general equilibrium model includes firms and households. We will start with firms in this section.

3.1 Firms

We model a firm as a collection of managers who coordinate on joint production. The firm-level production technology features increasing returns to the number of managers and complementarity across managers of heterogeneous skills. Non-family firms are constrained by a basic moral-hazard constraint: individual managers can steal a fraction of the joint output and forgo their managerial remunerations. The fraction that they may steal can be reduced by costly monitoring, which determines the optimal size of the firm. The limitation of family firms is that the size and the managerial skill endowment of a family are exogenously given and immutable.

Consider the following production function mapping the productivity vector of n

managers $\mathbf{z} = (z_1, ..., z_n)$ and workers $\mathbf{l} = (l_1, ..., l_n)$ in an organization into final output,

$$y = f(\mathbf{z}, \mathbf{l}) = n^{\alpha} \left[\frac{1}{n} \sum \left(z_i l_i^{\theta} \right)^{\rho} \right]^{1/\rho}$$

where $\rho < 1$. The complementarity between managers implies that organizations that only employ non-family managers will be perfectly sorted. $\theta \in [0, 1)$ is the span of control of an individual manager and $\alpha \geq 1$ governs the gains from specialization. We assume $\alpha + \theta < 2$.

Its worthwhile to emphasize some special cases of the production technology:

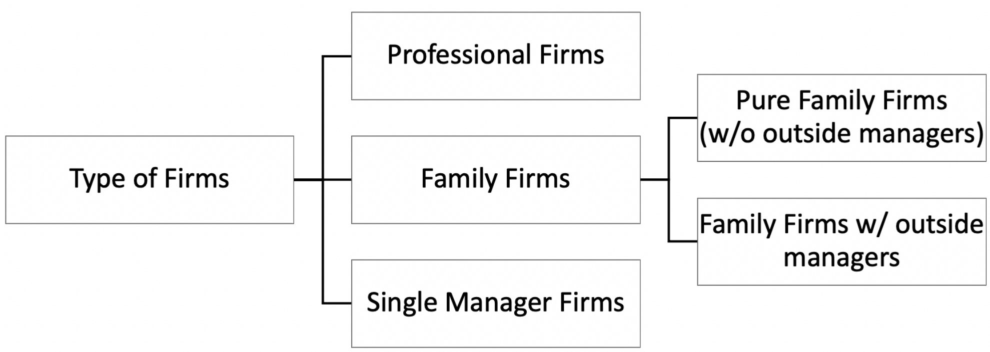
- Perfectly-sorted firm

$$f(\mathbf{z}, \mathbf{l}) = n^{\alpha} z l^{\theta}$$

- When there are no gains from specialization, i.e. $\alpha = 1$ or absent complementarities, i.e. $\rho = 1$:

$$f(\mathbf{z}, \mathbf{l}) = \sum_{i=1}^{n} z_i l_i^{\theta}$$

  In particular, note that there is no need to form a firm and the model simplifies to the standard span of control model.



We now describe various types of firms possible in this economy.

3.1.1 Perfectly-Sorted Non-Family Firm

The complementarity between managers implies that organizations that only employ non-family managers will be perfectly sorted[2]. We assume that managers can steal a

[2]see Appendix B.4

fraction ϕ/m of the output of the organization, where ϕ is a parameter and m is the per-manager monitoring. Therefore, payments to each individual manager τ must be at least as large as the income obtained by stealing the output,

$$\tau \geq \min\left\{1, \frac{\phi}{m}\right\} n^{\alpha} z, \tag{1}$$

and they must also be greater or equal to the market wage for their type of managers

$$\tau \geq w(z). \tag{2}$$

Noting that τ would never be greater than w(z), the problem of an organization with managers of talent z the level of monitoring per manager, m, and the total number of managers, n, is to maximize

$$\max_{n,l} n^{\alpha} z l^{\theta} - nm - nw(z) - wnl \tag{3}$$

$$\text{s.t.}$$

$$w(z) \geq \frac{\phi}{m}(n^{\alpha} z l^{\theta} - wnl) \tag{4}$$

Noting that the constraint would hold with equality, substituting m from the constraint in the objective function

$$\max_{n,l} \left[1 - \frac{n\phi}{w(z)}\right] \left(n^{\alpha} z l^{\theta} - wnl\right) - nw(z) \tag{5}$$

The first order conditions of this problem are

$$\alpha n^{\alpha-1} z l^{\theta} - w(z) - wl - \frac{\phi}{w(z)}(\alpha + 1) n^{\alpha} z l^{\theta} + \frac{\phi}{w(z)} 2nlw = 0, \tag{6}$$

$$\theta n^{\alpha} z l^{\theta-1} = wn, \tag{7}$$

Using zero profit entry condition, we know that equilibrium wage has to be,

$$w(z) = \left[1 - \frac{n\phi}{w(z)}\right] \left(n^{\alpha-1} z l^{\theta} - wl\right) \tag{8}$$

Solving (see Appendix B.1), we get,

$$
\begin{aligned}
l &= \left(\frac{n^{\alpha-1} z\theta}{w}\right)^{\frac{1}{1-\theta}} && (9)\\
n &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} && (10)\\
w(z) &= \frac{(\alpha-\theta)}{(\alpha-1)}\phi\left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} && (11)
\end{aligned}
$$

Note that wage function of professional managers is convex in z.

3.1.2 Family Firms

This section describes two types of family firms possible in this economy.

1. Pure Family Firms

We assume that family members cannot steal from the family firms. A pure family firm is therefore only constrained by the number of family members.

$$\max_{l} n^{\alpha}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho}\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w$$

Solving for optimal labor allocation,

$$n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(z l^{\theta})^{\rho}\right]\right\}^{1/\rho-1} \theta l_i^{\rho\theta-1} z_i^{\rho} = w \tag{12}$$

Which is nothing but the standard result of equalization of marginal product in such problems. Note in particular, that we can simplify this to get,

$$l_i = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \tag{13}$$

Where $\bar{z}(z,n) = \frac{1}{n}\sum_{i=1}^{n} z_i^{\frac{\rho}{1-\theta\rho}}$

Boundary case: One Person Family Firm

A special case of pure family firms is one person family firm or where $n = 1$

$$\pi^{1FF} = \max_{l} z_i l_i^{\theta} - l_i w$$

Thus, effective market wage for professional managers that the family firms with outside managers will take as given becomes:

$$w^e(z) = max(\pi^{1FF}, w(z)) \tag{14}$$

This gives us a simple cutoff z^{e*} between professional managers and one person firms.

2. Family Firm with Outside Managers

A family firm with n_f family members with (generalized) average ability z_f chooses the total number of managers $n \geq n_f$, the level of monitoring for outside managers m, and productivity of these managers z along with labor l_i to maximize,

$$\max_{m,n\geq n_f,z} n^{\alpha}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw - (n-n_f)cm - (n-n_f)w^e(z) \tag{15}$$

s.t.

$$w^e(z) \geq \frac{\phi}{m}\left(n^{\alpha}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right) \tag{16}$$

Following the argument for professional firms described earlier, the objective function becomes

$$\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\left(n^{\alpha}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right) - (n-n_f)w^e(z) \tag{17}$$

In addition to the incentive compatibility (henceforth IC) constraint for outside managers, family firms with outside managers are also subject to the family incentive compatibility constraint. The family IC constraint is that the family's joint profit when they don't steal has to be greater than a fraction of the profit when they collective steal and do not renegade the profit to outside managers. More formally, λ/m_f is the fraction they can collectively steal,

$$\pi^c \geq \frac{\lambda}{m_f}\pi^{nc}$$

where,

$$\pi^c = \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\left(n^\alpha\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho + (n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right) - (n-n_f)w^e(z) - n_f c m_f \quad (18)$$

$$\pi^{nc} = \left(n^\alpha\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho + (n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right) \quad (19)$$

Let $\pi^f = \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)$.
Thus, the problem becomes,

$$\max_{n,z,l} \frac{\pi^f(n,z,l) + \sqrt{\pi^f(n,z,l)^2 - 4\lambda\pi^{nc}(n,z,l)cn_f}}{2} \quad (20)$$

The domain of the above problem is restricted to: ${\pi^f}^2 \geq 4\lambda\pi^{nc}(n,z,l)cn_f$ and $\pi^f \geq 0$ It can be easily shown that the labor choice is not distorted because of family IC (see appendix B.2). Thus, we get,

$$l_i = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \quad (21)$$

Let the solution to max 20 be $n^*(n_f, z_f)$ and $z^*(n_f, z_f)$ which we are going to obtain numerically.

$$\pi^{nc} = \left(\frac{1}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} \left(\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right) \quad (22)$$

Where, $\bar{z}(z,n) = \left\{\frac{n_f}{n} z_f^{\frac{\rho}{1-\theta\rho}} + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}}\right\}$ where $z_f = \left(\frac{1}{n_f}\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}}\right)^{\frac{1-\theta\rho}{\rho}}$

3.1.3 Policy Functions

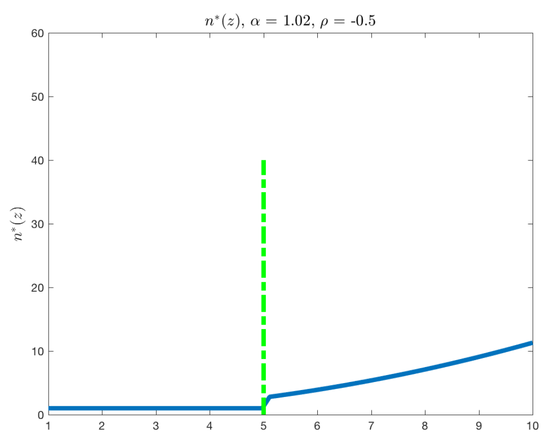
Professional Firms

The optimal number of managers in professional firms can be described as

$$n^{e*}(z) = \begin{cases} 1, & z \leq z^{e*} \\ n^*(z), & z > z^{e*} \end{cases} \tag{23}$$

A professional firm with z such that $z \leq z^{e*}$, becomes a single manager firm. On the other hand, professional firms operating with high z such that $z > z^{e*}$ have more than one managers. This is described in figure 5. Optimal workers per manager is analogous to optimal number of managers. As one would expect, optimal managers and workers per manager is an increasing function of productivity.

Figure 5: Policy Function: Professional Firms
$n^{e*}(z)$



Policy Functions: Family Firms

The optimal choice of outside managers,$n^*(n_f, z_f) - n_f$, is shown in 6 and their productivity $z^*(n_f, z_f) - z_f$ is shown in figure 7. If the effective z_f of the family firm is low, they do not hire any outside managers and are not affected by the incentive compatibility constraint that the family firms with outside managers will be subject to. For high enough z_f they hire outside managers and the number of outside managers is increasing in z_f. The number of managers they decide to hire increases marginally with n_f. Given complementarity in the production function and the wages that the firms face for professional managers, they hire managers with similar productivity as the family as clear from figure 7.

Figure 6: Policy Function: $n^*(n_f, z_f) - n_f$

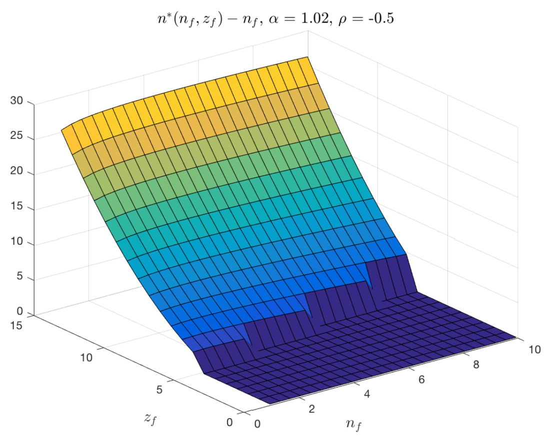
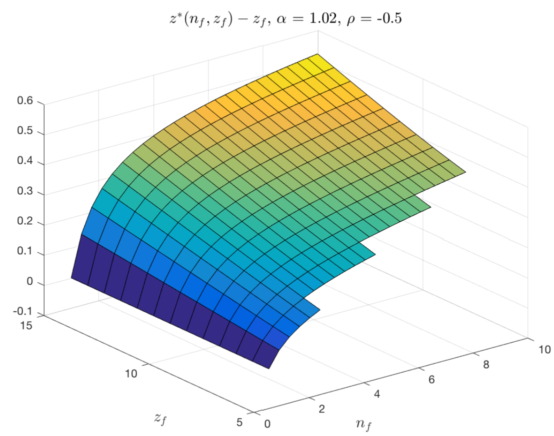


Figure 7: Policy Function: $z^*(n_f, z_f) - z_f$



3.2 Households

Individuals are born into families that differ in size and managerial skill endowment. We assume a unitary model of household with linear utility function where households collectively maximize household income. Therefore, each member of a family has the option to (i) work as a manager in the family firm; (ii) work as a manager in a non-family firm; or (iii) supply non-managerial labor for a wage. Given the family size, N and draws of z's $(z_1, z_2, ..., z_N)$, individuals jointly decide where to be a worker, professional manager, self-employed/ single manager firm or form a firm with other members of the family. Note that if they decide to form a family firm, they further decide whether to hire outside managers and workers to the firm, as described in the problem of family firm. In order to simplify the number of choices, we restrict the occupational choice to be at most one family firm per household.

Illustration: Family of Size 2

For a family of size 2 with z_1, z_2, the figure 8 illustrates the choices: if both z's are low they are both workers, if one is low while the other is high, the low z is a worker and the high z is a professional manager or managers a one person firm. If z's are along the diagonal, they decide to form a family firm, this result is because of complementarity in the production function. At this point, it is important to highlight the key tension in the model: while family members want to benefit from increasing returns technology and get around the contract enforcement constraint, they are constrained by the endowment of the family members- given the complementarities in the production function, they benefit from the increasing returns technology only if the cousins have similar productivity. When $z \leq z^{e*}$, individuals choose to run a one person firm. For clear exposition, we only provide four broad categories in the figure.

Figure 8: Occupational Choice: Family of Size 2

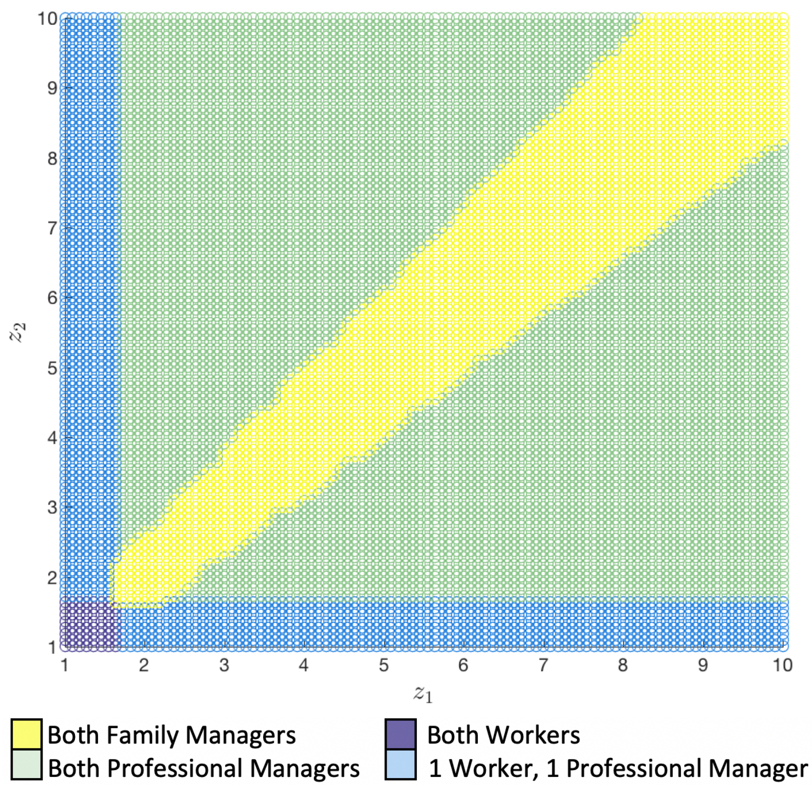


Illustration: Family of Size 3

Figure 10 represents the 3-D figure of occupational choice at the family level for family of size three. We also present a slice along the parallel line ($z_1 + z_2 + z_3 = 24$) in figure 9, this is a view along the diagonal of the cube. Note that for clear representation/ visualization of family firms, we club together workers, professional managers and one-person firms in this figure. When $z's$ along the main diagonal of the cube, they form a family firm of size 3 - policy function described in the previous section informs whether they hire outside managers or not: for low productivity along the main diagonal, they do not hire outside managers while along the top end of the main diagonal, they hire outside managers. Let's focus on the side formed by $z_2 = 1$ plane: as we saw in the family of size two, if the productivity is along this side diagonal, they form family firm of size two and the low productivity z_2 becomes a worker. Figure 10 illustrates this further- while along main diagonal there are family firms of size 3, if the productivity of one manager is doesn't align well with the other two family managers, the family is better off with her being a professional manager or running a firm by herself while the other two members form a family firm together. If the productivity of the other two managers also doesn't align with each other, the family is better off with all three being professional managers or running a one-person firm each (unless one's productivity is below the worker-professional cutoff in which case he opts to be a worker).

Figure 9: Occupational Choice: $n_f = 3$ (Simple)

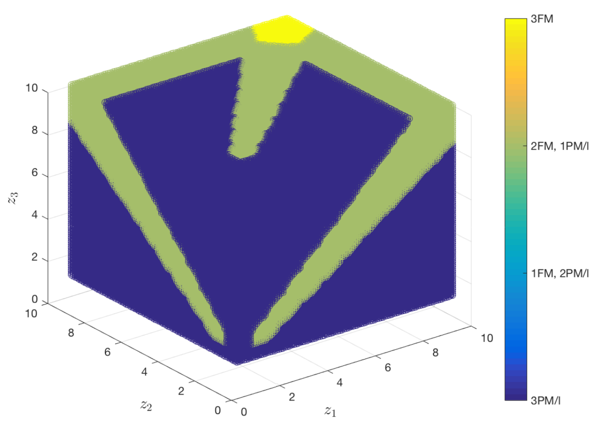
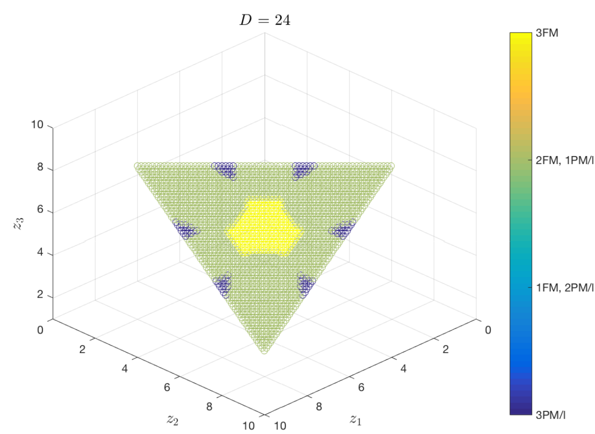


Figure 10: Slice along $z_1 + z_2 + z_3 = 24$



Equilibrium

The equilibrium can be described in the standard way. For given prices, i.e. worker wage w, effective manager wage w(z), aggregate labor demand equals supply from occupational choice at the family level. Note that we don't need to clear the managers market separately as long as there is a positive fraction of professional managers or one-person managers in the whole distribution above worker cutoff.

Algorithm: Labor Market Clearing

1. Start with a guess of w

2. Obtain $w^e(z)$ (explicitly, from professional firms and one person firm problem)

3. Solve for $n^*(z_f, n_f), z^*(z_f, n_f)$ (FF with outside managers and family IC)

4. Obtain the number of families for each size, N from data. Simulate multivariate lognormal $z's$ for N.

5. Solve occupational choice of the Family problem and obtain labor supply and family firm labor demand by keeping track of family firms

6. Obtain labor demand from professional firms by calculating $\int \frac{p(z)}{n(z)} n(z) l(z) F(dz)$, where $p(z)$ is fraction of professional managers among z types. We can calculate this by $p(z) = \frac{\sum_{N=1}^{N=10} N \text{ Number of professional managers } (z,N)}{\sum_{N=1}^{n_f=10} N \text{ Number } (z,N)}$

7. Obtain the demand for professional manager from family firms. If the demand for professional managers is less than supply, excess supply professional managers come together to form a professional firm.

8. If the demand for professional manager from family firms is greater than the professional managers supply, market wage $w^e(z)$ schedule adjusts wherein the managers for such z types seek rent from the family firms. In our baseline estimation for India, demand for professional managers by family firms is less than the supply of professional managers.

4. Data

We use Indian manufacturing data by putting together Annual Survey of Industries (ASI) and the Surveys of Unorganized Manufacturing conducted by the National Sample Survey Organization (NSS). We augment the establishment level data with CMIE Prowess along with household level data from IPUMS-India.

Survey of Unorganized Manufacturing by National Sample Survey (NSS) is a survey of establishments that employ $\leq$ 100. Its a 10% sample of the universe of Directory Manufacturing Establishments (DME) which employ a total of six or more workers, Non-directory Manufacturing Establishments (NDME) which employ total of five or less number of workers and Own Account Manufacturing Enterprises (OAME) which do not employing any hired workers. Annual Survey of Industries (ASI) is the establishment level census of manufacturing units employees ≥ 100 and covers about a third of. formal establishments with ≤ 100 employees for the year 1994-95. Both NSS and ASI gather similar information and put together they capture the entire distribution of employment in the manufacturing sector. See Hsieh and Klenow (2014) and Akcigit et al. (2016)

for a detailed description of ASI and NSS Datasets. We use employment, output and compensation information from NSS 1994-95 and ASI.

We obtain the family size distribution of India and the US by using the number of siblings from 0 to 18 years of age present in the household in the 1980s, who would represent the working population in our sample 20 years later. We use test scores of children within the household from ASER survey to discipline the correlation of skills at the household level.

Lastly, to perform our cross-country validation exercise, we obtain the micro-data from over 46 countries available in Global Entrepreneurship Monitor 2002-2005. We construct country level aggregates from individual level GEM survey data and merge it with output per worker using Penn World Table 9. We obtain the Contract Enforcement Score from World Bank Ease of Doing Business across countries present in GEM to compare our model estimates with the same.

Definition of a Family Firm

Throughout the paper, we call all the firms with atleast 2 members from the same family – involved with the active management of the firm – as family firms. We use this definition to abstract from ownership vs control vs management and use it consistently across data and in the model.

See Gopalan, Nanda, and Seru (2007) and Khanna and Palepu (2000) for a description on an alternative ownership and group affiliation based definitions.

5. Quantitative Exercise

Ability z is assumed multivariate $LN(\mu, \sigma, \psi)$, where μ and σ are the standard distribution parameters and ψ is the correlation within families. We assume the correlation matrix has equal off-diagonal terms, implying equal correlation in the ability of any two siblings.

5.1 Calibration

We use rich firm and household level micro-data from India to discipline the model. For family distribution, we use IPUMS India for 1983 and 1987 to measure the number of individuals less than 18 years of age who are siblings. We adjust the distribution by female labor force participation and allocate the fractions to the integers proportionally.

For example, if there are 100 families with 3.4 effective working members, we allocate 40 families to family size 3 and 60 to family size 4. Absent good family firms data which can help us identifying ρ and α, we do sensitivity around $\rho = -2.5$ and $\alpha = 1.03, 0.05, 1.07$. Our estimation strategy involves estimating parameters outside of the model and targeting moments to match moments within the model. In table 1, we describe the parameters estimated outside of the model. For firms with single managers, share of output going to labor, referred to as labor share, pins down θ. In order to obtain the correlation of productivities within families, ψ, we use the correlation between test scores of siblings using ASER data.

Table 1: Estimated outside of the Model

Moment	Source	Parameter	Value
Correlation of productivity within Family	ASER	ψ	0.49
Labor share, single manager firms	NSS-ASI	θ	0.34

The remaining parameters include the delegation friction and mean and scale parameters in the productivity distribution. We jointly target top concentration, average firm size and average number of managers to estimate the delegation friction, ϕ, and distribution parameters, μ and σ.

Table 2: Targeted Moments

Moment	$\alpha = 1.03$	$\alpha = 1.05$	$\alpha = 1.07$	Data	Source
Top 10-percentile employment share	0.37	0.32	0.35	0.43	NSSUM-ASI
Average Firm Size	3.06	2.99	3.10	2.42	NSSUM-ASI
Average Number of Managers	1.50	1.60	1.66	1.13	NSSUM-ASI

The resulting baseline calibration for different values of α is described in table 3.

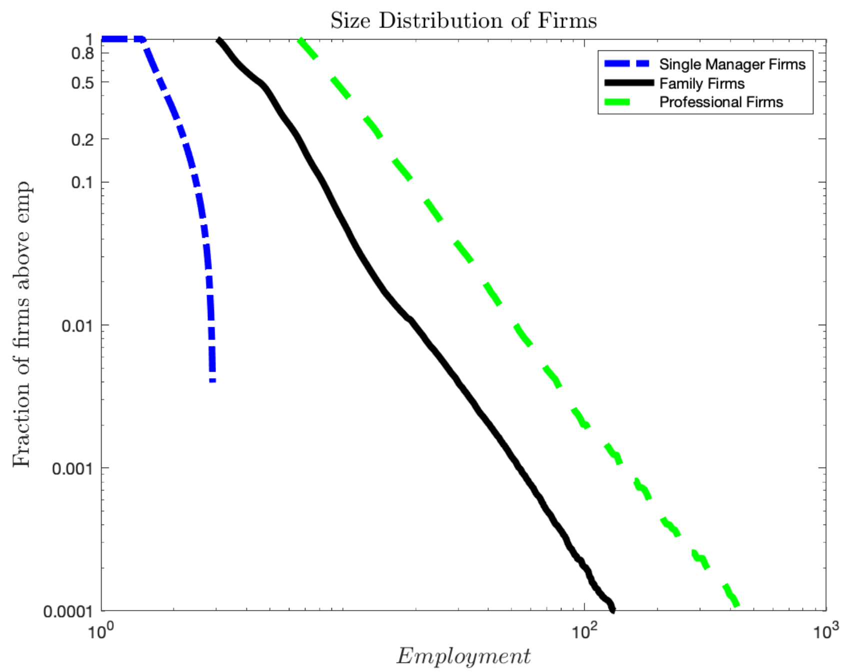
Table 3: Baseline Calibration

Parameter	$\alpha = 1.03$	$\alpha = 1.05$	$\alpha = 1.07$
ϕ	0.06	0.12	0.20
μ	1.94	2.18	2.16
σ	2.02	2.14	2.42

5.2 Size Distribution of Firms

Our estimated model gives rise to a rich set of firms. The model generates a large number lot of one person firms situated in the lower end of the distribution primarily as depicted in the in figure 11 which shows the size distribution of firms. This is because if a manager is productive enough ($z \geq z^{e*}$), their income as manager working in professional firm or family firms exceeds the profit earned by running a single firm. Family firms exist throughout the distribution and are on average less productive than one person firms and subsequently hire lower number of managers and workers than professional firms. A small number of professional firms exist in the upper end of the distribution but they are on average thrice as much productive compared to one person or family firms and employ multiple-folds more individuals per firm compared to their counterparts. It is also worthwhile to note that most family firms do not hire outside managers but the ones that hire are of very high productivity.

Figure 11: Size Distribution



The ranking of firms generated by the model lines up well with the (limited) data on size distribution we have in NSS Unorganized Manufacturing, which is limited to less than 100 employees and CMIE Prowess, which includes the publicly listed firms. We use the ownership category for NSS and last-names of Board of Directors to identify family firms, something we describe in detail in appendix.

Figure 12: Size Distribution Data: NSS

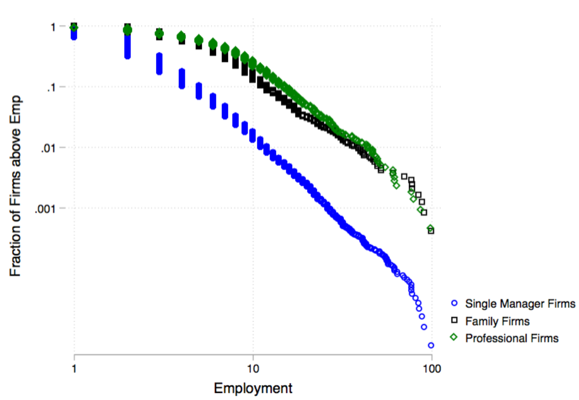
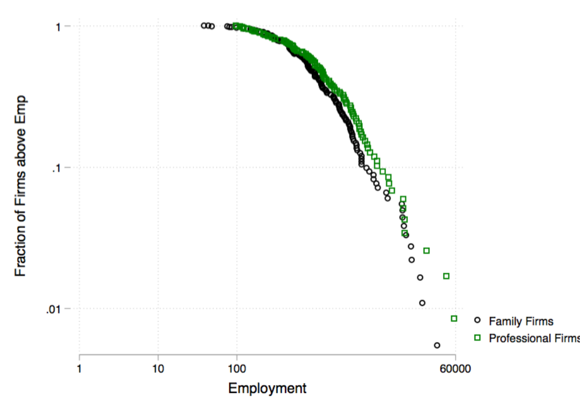


Figure 13: Size Distribution Data: Prowess



5.3 Delegation Friction across Countries

Given that we have estimated the structural parameters of the model on Indian data, we now use our calibrated model to quantify the differences in production and output per worker across countries. We assume that the countries differ in one key aspect of the model: contract enforcement or ϕ. Using micro-data of 46 countries present in GEM, Global Entrepreneurship Monitor, 2002-2005 survey (as in Poschke (2017)), we infer country level enforcement constraint ϕ, to match average firm size in each of the 46 countries present in our sample. Note that GEM is not a representative sample for manufacturing, which is our prime focus. Therefore, we use GEM to show the external validity of the model and use only India and US for output calculation, where we have data of manufacturing sector, in the results section.

We now show how the model predicted enforcement friction ϕ compares with the other external measures of contract enforcement across countries. We turn to the Contract Enforcement Score data from the World Bank Ease of Doing Business 2005. Note that in our model, higher value of enforcement friction ϕ corresponds to weaker rule of low, Contract Enforcement Score goes from 0-100 where a higher score is stricter rule of law and better contract enforcement. Figure 15 shows that our model predicted enforcement friction is highly correlated with Contract Enforcement Score from World Bank Ease of Doing Business. This provides some external validation of our model.

Figure 14: Delegation Friction (ϕ) Across Countries

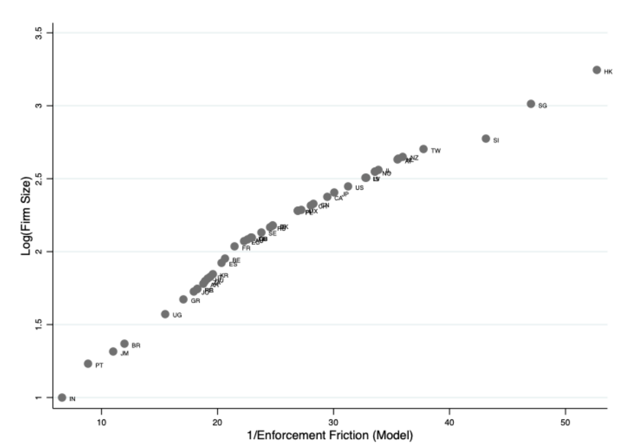
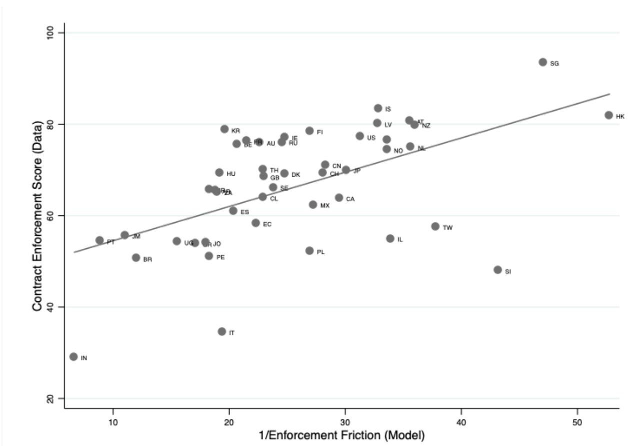


Figure 15: Delegation Friction (ϕ): Data vs Model



6. Results

Having estimated the model, we move to our quantitative exercises.

6.1 Role of Delegation Friction

Use cross-country differences in the average firm size across India (2.7) and the US (41.3) to measure differences in delegation frictions and quantify its impact on output. If India had the delegation efficiency of the US, its output per capita would go up by 6-16%, across various levels of α as described in table 4.

Table 4: Role of Delegation Friction

	$\alpha = 1.03$	$\alpha = 1.05$	$\alpha = 1.07$
% Change in India's Income	6.6%	11.5%	16.0%

Table 5: Untargeted Moment: Percentage of Family Firms

	$\alpha = 1.03$	$\alpha = 1.05$	$\alpha = 1.07$	Data
Family Firms in India ($\geq$ size 10)	52%	61%	70%	53% (WMS)
Family Firms in US ($\geq$ size 10)	1%	2%	4%	12% (WMS)

6.2 Value of Family Firms

In order to understand the role of family firm in the economy, we take away the option to form a family firm and show the implications for size distribution and income distribution. If family firms are not allowed in the model, income gap shown in the previous section increases by 14 to 20 percent. Not allowing family firms also leads to a missing middle from the firm size distribution and an increased market concentration as a result.

Table 6: Value of Family Firms

	$\alpha = 1.03$	$\alpha = 1.05$	$\alpha = 1.07$
% friction mitigated by Family Firms	13.9%	15.4%	20.3%

Figure 16: Filling the Middle

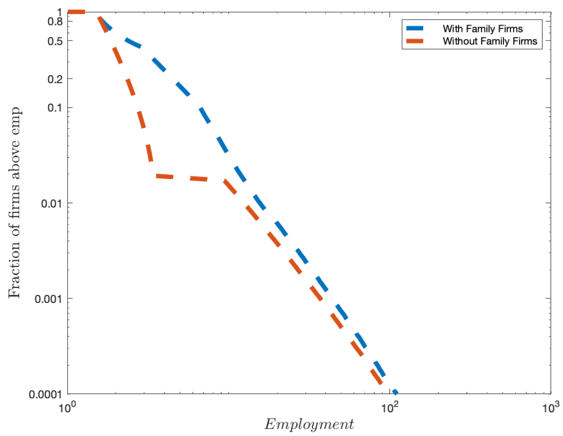
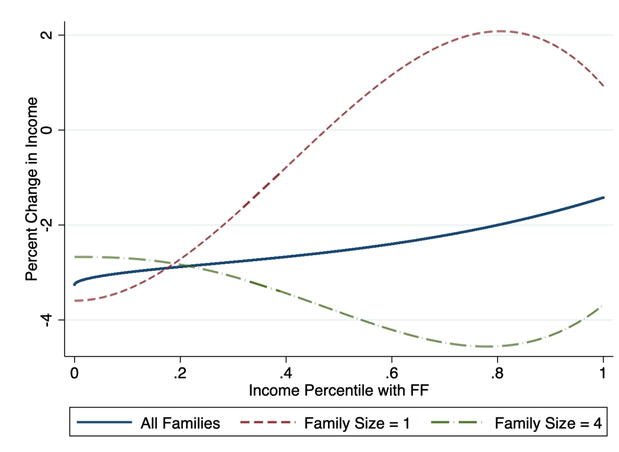


Figure 17: Distributional effect of family firms

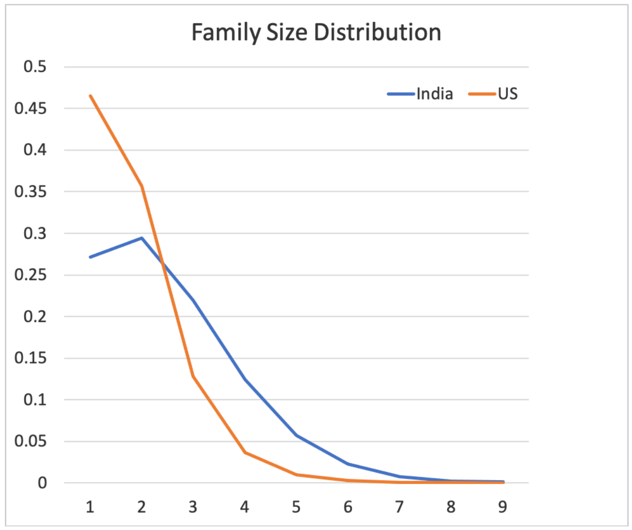


Wealthy small families gain 1-3% w/o family firms while poor small families lose upto 3.5%. The biggest losers are large wealthy families lose 1-4% w/o family firms who benefit most from the existence of family firms in presence of delegation frictions.

6.3 Role of Family Size

Lastly, we look at the role of family size distribution in mitigating these frictions. We feed in the estimated model the distribution from the US instead of India, as shown in figure 18 – keeping population fixed to abstract from the scaling effects. We find that if India had the same family distribution as the US, Income per capita goes down by 6-8% (50% of the distortion). Therefore, the ability to mitigate the friction goes down by half.

Figure 18: Family Size Distribution



Reducing family sizes *reduces* income per capita, which contrasts with the conventional wisdom on fertility and economic development.

7. Conclusion

This paper explores the mechanisms for the existence of family firms and its aggregate implications for economic development. We model a firm as a collection of managers who coordinate on joint production. The firm-level production technology features increasing returns to the number of managers and complementarity across managers of heterogeneous skills. Our model gives rise to the existence of a fairly rich set of firms: single person firms, professional firms, family firms without any outside managers and family firms with outside managers. In our model, the size of any non-family firm is limited by the ability of managers to divert a fraction of output, i.e., imperfect enforcement of contracts. On the other hand, size of a family firm is limited by the number of family members and their productivity endowment. Our quantitative model based on Indian micro data shows that India's income per capital would be 7 to 16 percent higher if contracts in India were enforced as well as in the US. If family firms are not allowed in the model, this income gap increases by 14 to 20 percent, since family firms are a way of mitigating the contractual frictions. Dissolving all family firms results in an income loss of 1 to 3 percent to large wealthy families and small poor families. In addition, the mid-range of the firm size distribution hollows out and income inequality worsens. A policy reducing family sizes undermines the role of family firms in mitigating the impact of contractual frictions and hence reduces income per capita, which contrasts with the conventional wisdom on fertility and economic development. Finally, family firms could play an important role in identifying the increasing returns and complementaries in production, as we show in the appendix. Something we leave for future work.

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Appendix

A. Data Appendix

A.1 Definition of Family Firm

World Management Survey

We identify family firm in World Management Survey[3] Manufacturing Questionnaire, which is a survey of the formal sector for various countries, by using the following question: "How many family members work in the management of the firm?" including the CEO. Any firm that has more or equal to two family members working in the management of the firm is labeled a family firm. Please see Bloom et al. (2014) for the details about the survey.

Prowess

Prowess compiles information from the stock exchange, annual report of companies and regulators and covers financial statements. It includes both public and privately held firms of the entire distribution of firm size and ownership categories. We use Prowess data from the year 2001 since the information on major shareholders and their share is available since 2001. Prowess also provides a "group affiliation" to identify firms belonging to groups such as Tata, Reliance etc. which claims to capture the complex ownership structure and controlling pattern. See Gopalan, Nanda, and Seru (2007) and Khanna and Palepu (2000) for a description on this. It also includes the names of board members and shareholders for companies and for listed firms, it also includes the compensation of directors and employees.

Throughout the paper, we call all the firms with atleast 2 members from the same family as family firms. In Prowess data, we use the uniqueness of lastname to identify whether two board members or shareholders are related. We call a firm a family firm if they have two individuals with the same lastname. There are two potential sources of error: first, family members not having the same lastname[4] and second, where two non-family members have the same lastname[5]. We thus take a random sample of firms

[3] https://worldmanagementsurvey.org/

[4] This can be with in-laws along with extended family members or names in southern India where sometimes lastname of the father typically is the firstname of the son. The latter we can incorporate in our algorithm. In that sense, this bias would underestimate the role family firms and thus our estimates can be considered as lower bounds.

[5] this is possible for some common lastnames

and see whether two individuals sharing the lastname are indeed related. We conclude that if two same-lastname individuals are on the board of directors, the error is $\leq 5\%$. We now describe our algorithm with two examples below.

Table 7: Example Family Firm: Essar Steel India Ltd.

Full Name	Lastname	Manual Search
Vikram Amin	Amin	
Jitender Balakrishan	Balakrishan	
G D Goswami	Goswami	
Jatinder Mehra	Mehra	
G A Nayak	Nayak	
Shashi Ruia	Ruia	Founder
Prashant Ruia	Ruia	Son of Shashi Ruia
Ravi Ruia	Ruia	Brother and Co-Founder
Sanjeev Shriya	Shriya	
S V Venkatesan	Venkatesan	
N B Vyas	Vyas	

Table 7 tabulates the list of board of directors in the year 2001 listed in prowess for Essar Steel India Ltd. Our algorithm identifies the lastname of everyone and notices three instances of lastname Ruia in the directors and thus categorizes Essar Steel into a family firm. A simple google search[6] points that brothers Shashi Ruia and Ravi Ruia started the firm and Prashant Ruia is the son of Sashi Ruia. Note that this manual search is possible only for top 1000 firms or so and we only take a small random sample to check our algorithm.

Table 8 describes the case of Siemens, a German Multinational Conglomerate. We see that none of the directors of Siemens share the same last name and thus our algorithm – rightly so – identifies Siemens as a non-Family Firm in our data.

A.2 Key Empirical Patterns

Prowess

We make the following observations from table 9,

1. 56% of the firms available in prowess are family firms. i.e. 56% of the firms either had two individuals from the same family in as active members of Board of Directors.

[6] https://en.wikipedia.org/wiki/Ravi_Ruia, https://www.weforum.org/people/prashant-ruia

Table 8: Example Non-family Firm. Siemens Ltd.

Full Name	Lastname
H Gelis	Gelis
Ashok P Jangid	Jangid
N J Jhaveri	Jhaveri
Y H Malegam	Malegam
F A Mehta (Dr.)	Mehta
A B Nadkarni	Nadkarni
O P Narula	Narula
O Schmitt (Dr.)	Schmitt
J Schubert	Schubert
D C Shroff	Shroff
Harminder Singh	Singh
S K Thackersey	Thackersey
P M Thampi	Thampi
K Wucherer (Dr.)	Wucherer

2. If we use Shareholders' based definition of family firm, the fraction of family firms goes to 72%, reflecting different channels through the families control the firm: through active management through the Board or through ownership as shareholders. For the purposes of this paper, we are going to take Directors' based definition since the model speaks to active managers of the firms.

Table 9: Prowess: Fraction of FF

	All
Family Firms	0.56

NSS

Unorganized manufacturing ownership distribution described in table 11 shows that about 97% of the firms in unorganized manufacturing are single person proprietary while 1.27% are family partnership and only 1.10% are non-family partnership.

In table 10, we provide summary stats by family size of the proprietor.
Firms with bigger family size are bigger in terms of value added, total employment, managers which is suggestive of a potential constraint for the firm. Firm managed by

Table 10: NSS: Summary Stats by Family Size

	0-3	4-6	6-8	≥ 8
Output	21617.47	35241.56	40000.95	68474.90
Age of ent. (yr.)	21.39	18.57	20.35	21.16
Total Full Time	1.80	2.25	2.51	3.48
Total Part Time	0.14	0.19	0.22	0.26

proprietors whose family has 8 or more people has three-fold output/ value added than those with a small family of less than 3 members.

NSS and IPUMS-India

Table 11: NSS: Ownership Distribution

	Unweighted	Weighted
Proprietory (male)	83.41	82.84
Proprietory (female)	13.44	14.72
Family Partnership	1.27	0.98
Non-Family Partnership	1.10	0.87
Co-operative	0.05	0.02
Public Sector	0.02	0.00
Limited	0.05	0.02
Other	0.66	0.54
N	192029	192029

NSS Provides us with ownership information of the establishment into single proprietary, family and non-family partnership as documented in table 11.

For the household level Information in IPUMS, among those who are self-employed in an household, we classify those self-employed in the same 3-digit industry, establishment size category and urban-rural classification as being in the same-firm[7]. With these definitions in mind, we now summarize some empirical patterns in the Indian data.

On ρ and α

Family firms data could be potentially useful in pinning down the production complementarities in standard class of models. In figure 19, we show that fraction of FF in top 1-ptile moves one-to-one with complementarity, ρ and increasing returns, $alpha$.While

[7]Ideally we would want to get some dataset to get a sense of how off we are

due to increasing returns individuals may want to form a family firm, if the production function exhibits more complementarity, they only do it with a similar in ability family member, thereby reducing the fraction of family firms.

Figure 19: On ρ: Fraction FF size $\geq$ 10

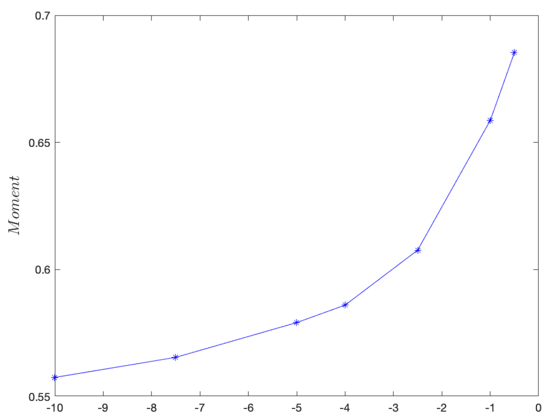
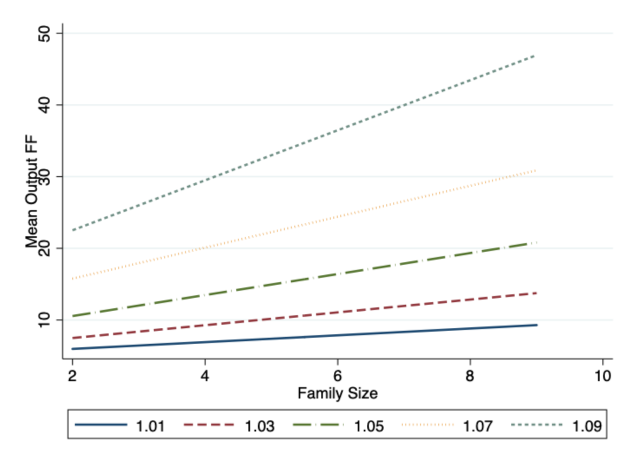


Figure 20: On α: Family Firm Output by Family Size



B. Proofs/ Algebra

B.1 Non-family Firm

$$\max_{n,l}\left[1-\frac{cn\phi}{w(z)}\right]\left(n^{\alpha}zl^{\theta}-wnl\right)-nw(z) \tag{24}$$

The first order conditions of this problem are

$$\alpha n^{\alpha-1}zl^{\theta}-w(z)-wl-c\frac{\phi}{w(z)}(\alpha+1)n^{\alpha}zl^{\theta}+\frac{c\phi}{w(z)}2nlw=0, \tag{25}$$

$$\theta n^{\alpha}zl^{\theta-1}=wn, \tag{26}$$

rearranging 26

$$\begin{aligned} l &= \left(\frac{n^{\alpha-1}z\theta}{w}\right)^{\frac{1}{1-\theta}} && (27) \\ & && (28) \end{aligned}$$

Equilibrium wage has to be,

$$w(z)=\left[1-\frac{cn\phi}{w(z)}\right]\left(n^{\alpha-1}zl^{\theta}-wl\right) \tag{29}$$

Subs 29 in 25

$$\begin{aligned} \alpha n^{\alpha-1}zl^{\theta}-wl-c\frac{\phi}{w(z)}(\alpha+1)n^{\alpha}zl^{\theta}+\frac{c\phi}{w(z)}2nlw-\left[1-\frac{cn\phi}{w(z)}\right]\left(n^{\alpha-1}zl^{\theta}-wl\right) &= 0 \\ \alpha n^{\alpha-1}zl^{\theta}-wl-c\frac{\phi}{w(z)}(\alpha+1)n^{\alpha}zl^{\theta}+\frac{c\phi}{w(z)}2nlw-\left[n^{\alpha-1}zl^{\theta}-wl-\frac{cn\phi}{w(z)}n^{\alpha-1}zl^{\theta}+\frac{cn\phi}{w(z)}wl\right] &= 0 \\ \alpha n^{\alpha-1}zl^{\theta}-wl-c\frac{\phi}{w(z)}(\alpha+1)n^{\alpha}zl^{\theta}+\frac{c\phi}{w(z)}2nlw-n^{\alpha-1}zl^{\theta}+wl+\frac{cn\phi}{w(z)}n^{\alpha-1}zl^{\theta}-\frac{cn\phi}{w(z)}wl &= 0 \end{aligned}$$

$$\alpha n^{\alpha-1}zl^{\theta}-c\frac{\phi}{w(z)}(\alpha)n^{\alpha}zl^{\theta}+\frac{c\phi}{w(z)}nlw-n^{\alpha-1}zl^{\theta} = 0 \tag{30}$$

Hence,

$$\begin{aligned}
\alpha n^{\alpha-1} z l^{\theta} - n^{\alpha-1} z l^{\theta} &= c\frac{\phi}{w(z)} \alpha n^{\alpha} z l^{\theta} - \frac{c\phi}{w(z)} n l w \\
(\alpha-1) n^{-1} z &= c\frac{\phi}{w(z)} \left(\alpha z - n^{1-\alpha} l^{1-\theta} w \right) \qquad (31)
\end{aligned}$$

Substituting 27,

$$(\alpha-1) n^{-1} z = c\frac{\phi}{w(z)} \left(\alpha z - n^{1-\alpha} \frac{n^{\alpha-1} z \theta}{w} w \right) \qquad (32)$$

Hence,

$$\begin{aligned}
w(z,n) &= c\frac{\phi n}{(\alpha-1)z} \left(\alpha z - \theta z \right) \\
w(z,n) &= \frac{(\alpha-\theta)}{(\alpha-1)} \phi c n \qquad (33)
\end{aligned}$$

Subs 33 in 29,

$$\begin{aligned}
\frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{c n \phi}{\frac{(\alpha-\theta)}{(\alpha-1)} \phi c n} \right] \left(n^{\alpha-1} z l^{\theta} - w l \right) \\
\frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{(\alpha-1)}{(\alpha-\theta)} \right] \left(n^{\alpha-1} z l^{\theta} - w l \right) \qquad (34)
\end{aligned}$$

Using 27, we note, $n^{\alpha-1} z l^{\theta} = \frac{lw}{\theta}$. Hence,

$$\begin{aligned}
\frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{(\alpha-1)}{(\alpha-\theta)} \right] \left(\frac{wl}{\theta} - w l \right) \\
\frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{(\alpha-1)}{(\alpha-\theta)} \right] \left(\frac{1}{\theta} - 1 \right) w l \qquad (35)
\end{aligned}$$

Substituting 27,

$$
\begin{aligned}
\frac{(\alpha-\theta)}{(\alpha-1)}\phi c n &= \left[\frac{(1-\theta)^2}{(\alpha-\theta)\theta}\right] w \left(\frac{n^{\alpha-1} z\theta}{w}\right)^{\frac{1}{1-\theta}} \\
n &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\theta\phi c}\right] w \left(\frac{n^{\alpha-1} z\theta}{w}\right)^{\frac{1}{1-\theta}} \\
n^{1-\frac{\alpha-1}{1-\theta}} &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\theta\phi c}\right] w^{1-\frac{1}{1-\theta}} (z\theta)^{\frac{1}{1-\theta}} \\
n^{\frac{2-\theta-\alpha}{1-\theta}} &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right] \left(\frac{w}{\theta}\right)^{-\frac{\theta}{1-\theta}} (z)^{\frac{1}{1-\theta}} \\
n &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} \qquad (36)
\end{aligned}
$$

Using 33, we get,

$$
w(z) = \frac{(\alpha-\theta)}{(\alpha-1)}\phi c \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} \qquad (37)
$$

B.2 Family Firm with Outside Managers: Part I

To show: The objective function leaves labor choice undistorted.

$$
\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^c - (n-n_f)w^e(z) - n_f c m_f \qquad (38)
$$

F.O.C. w.r.to. l_i,

$$
\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - n_f c \frac{\partial m_f}{\partial l_i} = 0 \qquad (39)
$$

m_f solves the following equation

$$
\begin{aligned}
\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z) - n_f c m_f &= \frac{\lambda}{m_f}\pi^{nc} \\
-\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} m_f + (n-n_f)w^e(z) m_f + n_f c m_f^2 &= -\lambda \pi^{nc} \\
n_f c m_f^2 - \left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right] m_f + \lambda \pi^{nc} &= 0
\end{aligned} \qquad (40)
$$

$$m_f = \frac{\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z) - \sqrt{\left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right]^2 - 4\lambda\pi^{nc}n_f c}}{2cn_f} \tag{41}$$

$$\begin{aligned}\frac{\partial m_f}{\partial l_i} = \frac{1}{2cn_f}\Bigg\{ \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - \frac{1}{2}\Bigg\{ \left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right]^2 - 4\lambda\pi^{nc}n_f c\Bigg\}^{-1/2} \\ 2\left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right]\left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - 4\lambda n_f c\frac{\partial \pi^{nc}}{\partial l_i}\right]\Bigg\}\end{aligned} \tag{42}$$

Substituting in the F.O.C.,

$$\begin{aligned}\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - \frac{1}{2}\Bigg\{ \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - \frac{1}{2}\Bigg\{ \left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right]^2 - \\ 4\lambda\pi^{c}n_f c\Bigg\}^{-1/2} 2\left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right] \\ \left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - 4\lambda n_f c\frac{\partial \pi^{c}}{\partial l_i}\right]\Bigg\} = 0\end{aligned}$$

We can take the partial out to write,

$$\begin{aligned}\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right) - \frac{1}{2}\Bigg\{ \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right) - \frac{1}{2}\Bigg\{ \left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right]^2 - \\ 4\lambda\pi^{c}n_f c\Bigg\}^{-1/2} 2\left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)\right] \\ \left[\left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right) - 4\lambda n_f c\right]\Bigg\}\frac{\partial \pi^{nc}}{\partial l_i} = 0\end{aligned}$$

This implies,

$$\frac{\partial \pi^{nc}}{\partial l_i} = 0$$

Thus, the labor choice is not distorted!

B.3 Family Firm with Outside Managers: Part II

Let $\pi^f = \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)$.

Thus, the problem becomes,

$$\max_{n,z,l} \frac{\pi^f(n,z,l) + \sqrt{\pi^f(n,z,l)^2 - 4\lambda\pi^{nc}(n,z,l)cn_f}}{2} \tag{43}$$

Important note: the domain is restricted to: ${\pi^f}^2 \geq 4\lambda\pi^{nc}(n,z,l)cn_f$ and $\pi^f \geq 0$

Solving for optimal labor allocation,

$$n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho + (n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho-1}\theta l_i^{\rho\theta-1}z_i^\rho = w \tag{44}$$

and

$$n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho + (n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho-1}\theta l^{\rho\theta-1}z^\rho = w \tag{45}$$

Simplifying 44 and 45, we get,

$$l_i = l_1\left(\frac{z_i}{z_1}\right)^{\frac{\rho}{1-\rho\theta}} \tag{46}$$

Subs 46 in 44,

$$\begin{aligned}
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^\rho l_1^{\theta\rho}\left(\frac{z_i}{z_1}\right)^{\frac{\theta\rho^2}{1-\rho\theta}} + (n-n_f)z^\rho l_1^{\theta\rho}\left(\frac{z}{z_1}\right)^{\frac{\theta\rho^2}{1-\rho\theta}}\right]\right\}^{1/\rho-1}\theta l_1^{\rho\theta-1}z_1^\rho &= w \\
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} l_1^{\theta\rho} z_1^{-\frac{\theta\rho^2}{1-\rho\theta}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}} l_1^{\theta\rho} z_1^{-\frac{\theta\rho^2}{1-\rho\theta}}\right]\right\}^{1/\rho-1}\theta l_1^{\rho\theta-1}z_1^\rho &= w \\
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}}\right]\right\}^{1/\rho-1} l_1^{\theta-1} z_1^{-\frac{\theta\rho(1-\rho)}{(1-\rho\theta)}+\rho}\theta &= w \\
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}}\right]\right\}^{1/\rho-1} l_1^{\theta-1} z_1^{\frac{\rho(1-\theta)}{(1-\rho\theta)}}\theta &= w
\end{aligned} \tag{47}$$

Let $\bar{z}(z,n) = \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} + (n-n_f) z^{\frac{\rho}{1-\theta\rho}} \right] \right\}$.
or
$\bar{z}(z,n) = \left\{ \frac{n_f}{n} z_f + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}} \right\}$ where $z_f = \frac{1}{n_f} \sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}}$
Thus,

$$n^{\alpha-1} \bar{z}(z,n)^{1/\rho-1} l_1^{\theta-1} z_1^{\frac{\rho(1-\theta)}{(1-\rho\theta)}} \theta = w \tag{48}$$

or

$$\begin{aligned} l_1^{1-\theta} &= \frac{\theta}{w} z_1^{\frac{\rho(1-\theta)}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho}} n^{\alpha-1} \\ l_1 &= \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_1^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \end{aligned} \tag{49}$$

More generally,

$$l_i = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \tag{50}$$

Let the solution to max 43 be $n^*(n_f, z_f)$ and $z^*(n_f, z_f)$.
Subs 50 in π^{nc}

$$\left(n^{\alpha} \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^{\theta})^{\rho} + (n-n_f)(z l^{\theta})^{\rho} \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f) l w \right) \tag{51}$$

$$
\begin{aligned}
l_i &= \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \\
l_i^{\theta} &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} z_i^{\frac{\rho\theta}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{\rho(1-\theta)}} n^{\frac{(\alpha-1)\theta}{1-\theta}} \\
z_i l_i^{\theta} &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} z_i^{\frac{1}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{\rho(1-\theta)}} n^{\frac{(\alpha-1)\theta}{1-\theta}} \\
(z_i l_i^{\theta})^{\rho} &= \left(\frac{\theta}{w}\right)^{\frac{\rho\theta}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{1-\theta}} n^{\frac{(\alpha-1)\theta\rho}{1-\theta}} \\
\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right] &= \left(\frac{\theta}{w}\right)^{\frac{\rho\theta}{1-\theta}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{1-\theta}} n^{\frac{(\alpha-1)\theta\rho}{1-\theta}} \bar{z}(z,n) \\
\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right] &= \left(\frac{\theta}{w}\right)^{\frac{\rho\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{1-\theta}} n^{\frac{(\alpha-1)\theta\rho}{1-\theta}} \\
\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right]\right\}^{1/\rho} &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{(\alpha-1)\theta}{1-\theta}} \\
n^{\alpha}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right]\right\}^{1/\rho} &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}}
\end{aligned}
$$

Thus,

$$
\pi^{nc} = \left(n^{\alpha}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^{\theta})^{\rho} + (n-n_f)(zl^{\theta})^{\rho}\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right)
$$

$$
\begin{aligned}
&= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} - w\left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} \\
&= \left(\frac{1}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} \left(\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right) \qquad (52)
\end{aligned}
$$

$\bar{z}(z,n) = \left\{\frac{n_f}{n} z'_f + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}}\right\}$ where $z'_f = \frac{1}{n_f}\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}}$

For clear interpretation, lets relabel stuff: $z'_f = (z_f)^{\frac{\rho}{1-\theta\rho}}$ or $z_f = (z'_f)^{\frac{1-\theta\rho}{\rho}}$

Thus, $\bar{z}(z,n) = \left\{\frac{n_f}{n} z_f^{\frac{\rho}{1-\theta\rho}} + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}}\right\}$ where $z_f = \left(\frac{1}{n_f}\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}}\right)^{\frac{1-\theta\rho}{\rho}}$

B.4 Professional Firms

Lets say the professional firm of type z wants to deviate and hire an ϵ number of managers of type z_1.

$$\pi(\epsilon) = \left(1 - \frac{n\phi}{w(z)} - \frac{\epsilon\phi}{w(z_1)}\right)\left((n+\epsilon)^{\alpha}\left\{\frac{1}{n+\epsilon}\left[nz^{\rho} + \epsilon z_1^{\rho}\right]\right\}^{1/\rho}\right) - nw(z) - \epsilon w(z_1) \tag{53}$$

$$n = \left[\frac{\alpha-1}{\alpha^2}\frac{1}{c\phi}z\right]^{\frac{1}{2-\alpha}} \tag{54}$$

$$\frac{\partial\pi(\epsilon)}{\partial\epsilon} = \frac{-\phi}{w(z_1)}(n+\epsilon)^{\alpha-\frac{1}{\rho}}\left[nz^{\rho} + \epsilon z_1^{\rho}\right]^{1/\rho} - w(z_1) + \left(1 - \frac{n\phi}{w(z)} - \frac{\epsilon\phi}{w(z_1)}\right)$$
$$\left[\alpha - \frac{1}{\rho}(n+\epsilon)^{\alpha-\frac{1}{\rho}-1}[nz^{\rho} + \epsilon z_1^{\rho}]^{1/\rho} + (n+\epsilon)^{\alpha-\frac{1}{\rho}}\frac{z^{\rho}}{\rho}[nz^{\rho} + \epsilon z_1^{\rho}]^{1/\rho-1}\right] \tag{55}$$

$$\frac{\partial\pi(\epsilon)}{\partial\epsilon} = (n+\epsilon)^{\alpha-\frac{1}{\rho}}\left[nz^{\rho} + \epsilon z_1^{\rho}\right]^{1/\rho}\left\{\frac{-\phi}{w(z_1)} + \left(1 - \frac{n\phi}{w(z)} - \frac{\epsilon\phi}{w(z_1)}\right)\right.$$
$$\left.\left[\left(\alpha - \frac{1}{\rho}\right)(n+\epsilon)^{-1} + \frac{z_1^{\rho}}{\rho}[nz^{\rho} + \epsilon z_1^{\rho}]^{-1}\right]\right\} - w(z_1) \tag{56}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = n^{\alpha-\frac{1}{\rho}}\left[nz^{\rho}\right]^{1/\rho}\left\{\frac{-\phi}{w(z_1)} + \left(1 - \frac{n\phi}{w(z)}\right)\right.$$
$$\left.\left[\left(\alpha - \frac{1}{\rho}\right)n^{-1} + \frac{z_1^{\rho}}{\rho}[nz^{\rho}]^{-1}\right]\right\} - w(z_1) \tag{57}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = n^{\alpha}z\left\{\frac{-\phi}{w(z_1)} + \left(1 - \frac{n\phi}{w(z)}\right)\right.$$
$$\left.\left[\left(\alpha - \frac{1}{\rho}\right)n^{-1} + \frac{z_1^{\rho}}{\rho}[nz^{\rho}]^{-1}\right]\right\} - w(z_1) \tag{58}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = n^{\alpha}z\left\{\frac{-\phi}{w(z_1)} + \left(1 - \frac{n\phi}{w(z)}\right)\left[\left(\alpha - \frac{1}{\rho}\right)\frac{1}{n} + \frac{1}{\rho n}\left(\frac{z_1}{z}\right)^{\rho}\right]\right\} - w(z_1) \tag{59}$$

Subs n and $w(z)$ and noting $k = \frac{\alpha-1}{\alpha}\frac{1}{\phi}$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha}{2-\alpha}}z^{\frac{2}{2-\alpha}}\left\{-\alpha\phi k^{\frac{\alpha-1}{\alpha-2}}z_1^{\frac{1}{\alpha-1}} + \frac{k^{\frac{1}{\alpha-2}}z^{\frac{1}{\alpha-2}}}{\alpha}\left[\left(\alpha-\frac{1}{\rho}\right)+\frac{1}{\rho}\left(\frac{z_1}{z}\right)^{\rho}\right]\right\} - \frac{1}{\alpha}k^{\frac{\alpha-1}{2-\alpha}}z_1^{\frac{1}{2-\alpha}} \tag{60}$$

Let $\lambda = \frac{z_1}{z}$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha-1}{2-\alpha}}z^{\frac{1}{2-\alpha}}\left\{-\alpha\phi k\lambda^{\frac{1}{\alpha-2}} + \frac{1}{\alpha}\left(\alpha-\frac{1}{\rho}+\frac{\lambda^{\rho}}{\rho}-\lambda^{\frac{1}{2-\alpha}}\right)\right\} \tag{61}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha-1}{2-\alpha}}z^{\frac{1}{2-\alpha}}\left\{1+\frac{\lambda^{\rho}}{\alpha\rho}-\left(\frac{1}{\alpha\rho}+\frac{\lambda^{\frac{1}{2-\alpha}}}{\alpha}+\alpha\phi k\lambda^{\frac{1}{\alpha-2}}\right)\right\} \tag{62}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha-1}{2-\alpha}}z^{\frac{1}{2-\alpha}}\left\{1+\frac{\lambda^{\rho}}{\alpha\rho}-\left[\frac{1}{\alpha\rho}+\frac{\lambda^{\frac{1}{2-\alpha}}}{\alpha}+\left(1-\frac{1}{\alpha}\right)\lambda^{\frac{1}{\alpha-2}}\right]\right\} \tag{63}$$

Taking the term inside brackets,

$$\pi_1(\lambda) = \left\{1+\frac{\lambda^{\rho}}{\alpha\rho}-\left[\frac{1}{\alpha\rho}+\frac{\lambda^{\frac{1}{2-\alpha}}}{\alpha}+\left(1-\frac{1}{\alpha}\right)\lambda^{\frac{1}{\alpha-2}}\right]\right\} \tag{64}$$

Now, we want to show that $\pi_1(\lambda) \leq 0$. If we find the λ_{max} associated with max of $\pi_1(\lambda)$ and show that at that $\pi_1(\lambda_{max}) \leq 0$, we are done(?)

$$\frac{\partial\pi_1(\lambda)}{\partial\lambda} = \frac{\lambda^{\rho-1}}{\alpha} - \frac{\lambda^{\frac{\alpha-1}{2-\alpha}}}{\alpha(2-\alpha)} - \frac{(\alpha-1)\lambda^{\frac{3-\alpha}{\alpha-2}}}{\alpha(\alpha-2)} \tag{65}$$

$\left.\frac{\partial\pi_1}{\partial\lambda}\right|_{\lambda=1} = 0$ and $\left.\frac{\partial^2\pi_1}{\partial\lambda^2}\right|_{\lambda=1} < 0$

$$\pi_1(1) = \left\{1+\frac{1}{\alpha\rho}-\left[\frac{1}{\alpha\rho}+\frac{1}{\alpha}+\left(1-\frac{1}{\alpha}\right)\right]\right\} = 0 \tag{66}$$

Thus,

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} \leq 0 \tag{67}$$